A

18731 **120 MINUTES**

1. The value of $\sum_{n=1}^{\infty} \frac{(2^{n+2} - 3^{n+1})}{n}$ is

   A) 6 B) 3 C) 0 D) ∞

2. $\lim_{n\to\infty} \frac{1+\sqrt{2}+\sqrt[3]{3}+...+\sqrt[n]{n}}{n}$

   A) equals ∞ B) equals 0 C) equals 1 D) does not exist

3. The series $\sum_{n=1}^{\infty} n^{10000} x^n$ converges absolutely when

   A) $x < 1$ B) $x > 1$ C) $|x| > 1$ D) $|x| < 1$

4. Any finite subset of a metric space is

   A) Open B) Closed
   C) Not necessarily open D) Not necessarily closed

5. Which of the following statement(s) is/are true in general?

   I: Any infinite set contains a countable set.

   II: Any infinite subset of a countable set is uncountable.

   A) I alone B) II alone
   C) Neither I nor II D) Both I and II

6. Which of the following forms a basis set for the vector space $\Re^3$?

   A) (1, 2, 1), (1,-2, 5) and (3, -2, 11)
   B) (1, 3, 1), (1, 0, -1), (3, -1, 0) and (2, 1,-2)
   C) (1, 1, 2), (1, 2, 5) and (5, 3, 4)
   D) (1, 1, 1), (1, 2, 3) and (2, -1, 1)

7. If U and W are finite dimensional subspaces of a vector space V, then dim (U+W) is

   A) dim (U) + dim (W) B) < dim (U) + dim (W)
   C) > dim (U) + dim (W) D) $\geq$ dim (U) + dim (W)

8. Let V be the subspace of $\Re^4$ generated by the vectors (1,-2,5,-3), (2,3,1,-4) and (3,8,-3,-5). Then the dimension of V is

   A) 4 B) 3 C) 2 D) 1

9. Let A be any **m x n** matrix. Then the trace of AA^T is the

   A) sum of all the elements in A
   B) sum of squares of all the elements in A
   C) sum of squares of all the diagonal elements in A
   D) square of the sum of diagonal elements in A

10. Inverse of a lower triangular matrix is

A) Lower triangular B) Upper triangular

C) A diagonal matrix D) A tridiagonal matrix

11. Which of the following statement(s) is/are true?

I: If A is idempotent and A≠I, then A^{-1} does not exist.

II: Sum of two skew symmetric matrices is skew symmetric.

A) I alone B) II alone C) Neither I nor II D) Both I and II

12. Which of the following statement(s) is/are true?

I: Every subsets of $\Re$ are Borel sets

II : Every Borel sets are not measurable

A) I alone B) II alone C) Neither I nor II D) Both I and II

13. Given $f(x) = \frac{1}{2}\chi_{[0,3]}(x), x \in R$ where $\chi_A(x)$ is the usual indicator function defined by

$\chi_A(x) = 1, \ x \in A$

$= 0, \ x \notin A$

Then state which of the following is the Lebeugue integral of f ?

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) 3 D) 0

14. For any two events A and B, P(A – B) is

A) P(A) – P(B) B) P(A) – P(AB)

C) P(A) $P(B^c)$ D) P(A) – $P(B^c)$

15. If two events A and B each with positive probabilities are independent, then they are

A) Mutually exclusive

B) Always mutually exclusive

C) Always mutually exclusive and equally likely

D) Always not mutually exclusive

16. The total number of conditions for the mutual independence of five events is:

A) 24 B) 26 C) 28 D) 32

17. Let E_1 and E_2 be any two events, then:

A) $P(E_1 \cup E_2) \le P(E_1)$

B) $P(E_1 \cup E_2) \le P(E_2)$

C) $P(E_1 \cup E_2) \le \min\{P(E_1), P(E_2)\}$

D) $P(E_1 \cup E_2) \ge \max\{P(E_1), P(E_2)\}$

18. Suppose X is a non-negative random variable with finite expectation, then which of the following is true?

A) $E(\log X) \le \log E(X)$ B) $E(\log X) \ge \log E(X)$

C) $E(\log X) = \log E(X)$ D) All of the above

19. Suppose X follows Binomial $b(10, \frac{1}{2})$ distribution, then:

A) $P(X < 5) = 1/2$

B) $P(X > 5) = 1/2$

C) $P(X < 5) \leq 1/2$

D) $P(X > 5) \geq 1/2$

20. If X and Y are independent random variables such that $E(X/(X+Y)) = 2$, then $E(Y/(X+Y))$ is :

A) -1 B) 1 C) 2 D) 0.5

21. If $X \geq 0$ then $\int_0^\infty P(X > x)\,dx$ is:

A) $E(X)$

B) $V(X)$

C) $E|X|$

D) Median of X

22. Let $X_1, X_2, X_3, X_4, X_5, X_6$ be the random variables such that $Var(X_i) = 4$ and $Cov(X_i, X_j) = 3$, i, j=1,2,...6 and i<j then $Var(X_1 - X_2 + X_3 - X_4 + X_5 - X_6)$ is:

A) 3 B) 4 C) 6 D) 10

23 Which of the following is not a distribution function:

A) $F(x) = e^{-e^{-x}}$;xϵR

B) $F(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$

C) $F(x) = \begin{cases} (1/2)e^x, & x < 0 \\ 1 - (\frac{1}{2})e^{-x}, & x \geq 0 \end{cases}$

D) $F(x) = \begin{cases} (1-\epsilon)/(1+e^{-x}), & x < 0 \\ \epsilon + (\frac{1-\epsilon}{1+e^{-x}}), & x \geq 0 \end{cases}$

24. Let $F(x)$ and $G(y)$ be the distribution functions of the continuous random variables X and Y. Then the value of $E[2F(X) + (1/2)G(Y)]$ is

A) 4/5 B) 5/4 C) 2/3 D) 3

25. If X follows $N(0,1)$ .What is $E|X|$?

A) 0 B) 1 C) $\sqrt{(\pi/2)}$ D) $\sqrt{2}/\sqrt{\pi}$

26. For all the pairs $(x_1, y_1), (x_2, y_2); x_1 < x_2$ and $y_1 < y_2$ which of the following is true for a two dimensional random variable?

A) $F(x_1, y_1) + F(x_2, y_2) + F(x_1, y_2) + F(x_2, y_1) \geq 0$

B) $F(x_1, y_1) - F(x_2, y_2) - F(x_1, y_2) + F(x_2, y_1) \geq 0$

C) $F(x_1, y_1) + F(x_2, y_2) - F(x_2, y_1) - F(x_1, y_2) \geq 0$

D) $-F(x_1, y_1) - F(x_2, y_2) + F(x_2, y_1) + F(x_1, y_2) \geq 0$

27. The characteristic function of a random variable X is :

$$\frac{1}{10}e^{-it}(2+e^{it}+4e^{i2t}+3e^{i3t})$$

Then $P\{2X+5<7\}$ is:

A) 3/10 B) 7/10 C) 1 D) 4/10

28. Let $P(.)$ be the pgf of a random variable X . Then pgf of mX is:

A) $mP(s)$

B) $P(s^m)$

C) $sP(m)$

D) $P(ms)$

29. If $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{L} C$, then:

A) $X_n + Y_n \xrightarrow{L} X + C$

B) $X_n + Y_n \xrightarrow{P} X + C$

C) $g(X_n)$ doesn't converge in law to $g(X)$

D) $X_n - Y_n$ doesn't converge in law to $X - C$

30. If $X_n \xrightarrow{P} 0$, then, $\lim_{n\to\infty} E\left\{\frac{|X_n|}{1+|X_n|}-1\right\} =$

A) 0 B) 1 C) -1 D) ∞

31. Let X follows Binomial distribution b(5,1/3). Then the point at which $P(X=x)$ is maximum:

A) 2 B) 3 C) 4 D) All of the above

32. Let X_1 and X_2 are independent binomial random variables following b(n,1/2) and b(m,1/2) respectively. Then which of the following is true?

A) $X_1 - X_2$ is binomial

B) $2X_1$ is binomial

C) $X_2 - X_1 + n$ is binomial

D) $nX_1 + mX_2$ is binomial

33. If X_1 and X_2 are independent Poisson random variables then, the conditional distribution of $X_1|(X_1+X_2)$ is

A) Poisson B) Binomial C) Geometric D) Hypergeometric

34. Which of the following is not true?

A) $E(X) = V(X)$, if X is Poisson

B) All cumulants of a Poisson distribution are equal to the parameter of the distribution

C) $\frac{(X-\lambda)}{\sqrt{\lambda}} \to N(0.1)$ as $\lambda \to \infty$, if X is Poisson

D) Poisson distribution is bimodal

35. Let X be a random variable whose pmf is $P\{X = x\} = (1/3).(2/3)^x; x = 0,1,2,...$

Then $P\{X = 11 | X > 10\}$ is:

A) 1/3 B) 2/3 C) $(1/3)^{10}$ D) $(2/3)^{10}$

36. Which of the following distributions obey memorylessness property?

A) Uniform B) Geometric C) Gamma D) Normal

37. If X follows exponential distribution with parameter λ. Then what is $E(e^{-\lambda x})$:

A) ½ B) 1 C) 0 D) -1

38. If X and Y are independent and identically distributed random variables following Uniform distribution U (-a,a). Then which of the following is true?

A) $X + Y$ is Uniform

B) $X - Y$ is Uniform

C) $X + Y \overset{d}{=} X - Y$

D) $X + Y$ is independent of $X - Y$

39. Which of the following is not true in the case of normal distribution?

A) If X and Y are iid Normal, then X+Y and X-Y are independent

B) $X / Y \overset{d}{=} X / (|Y|)$

C) If X and Y are independent and X+Y and X-Y are independent, then X and Y are normal.

D) If X and Y are standard normal variates then, $X^2 + Y^2 \sim \chi^2_{(2)}$.

40. Suppose X follows N(0,1) and Y follows Normal distribution N(1,2). Then,

A) $P(X > 0) = P(Y > 1)$

B) $P(X > 0) = P((Y/2) > 1)$

C) $P(X > 0) = P(Y > (1/\sqrt{2}))$

D) $P(X > 0) = P(Y > 0)$

41. If X and Y are independent $\chi^2_{(m)}$ and $\chi^2_{(n)}$, then the distribution of $Z = \frac{X}{Y}$ is

A) χ^2 B) Normal

C) Beta type I D) Beta type II

42. X follows $N(\mu,\sigma^2)$, then e^X has

A) Normal with mean e^{μ} and variance $e^{2\sigma^2}$

B) Exponential with mean e^{μ} and variance $e^{2\sigma^2}$

C) Lognormal with mean μ and variance σ^2

D) Lognormal with parameters μ and σ

43. The regression lines of Y on X and X on Y are $Y = aX + b$ and $X = cY + d$. The ratio of the S.D of X and Y is:

A) a/c B) c/a C) √(c/a) D) √(a/c)

44. Which of the following is not true in the case of bivariate normal distribution?

A) Marginal distribution of bivariate normal distribution is univariate normal.

B) If X_1 and X_2 are univariate normal then their joint distribution is bivariate normal.

C) If $Cov(X_1, X_2) = 0$, then X_1 and X_2 are independent.

D) All are not true.

45. The random variables X and Y such that $X+Y$ and $X-Y$ are positively correlated. Then,

A) $V(X) < V(Y)$

B) $V(X) > V(Y)$

C) $V(X+Y) > V(X-Y)$

D) $V(X+Y) < V(X-Y)$

46. Let X and Y be independent N(0,1) random variables. The distribution of $Z = ((X-Y)/(X+Y))^2$ is:

A) $\chi^2_{(1)}$

B) $\chi^2_{(2)}$

C) $F(1,1)$

D) $F(2,2)$

47. X_1 and X_2 are independent Poisson random variables with parameter λ, an unbiased estimator of λ^2 is :

A) $(X_1^2 + X_2^2)/2$

B) $(X_1 + X_2)/2$

C) $Max\{X_1, X_2\}$

D) $(X_1^2 + X_2^2 - (X_1 + X_2))/2$

48. Let p be the proportion of defective items produced by a machine. Suppose n items produced by the machine are examined and a random variable X_i is defined as

$X_i = 1$, if the i-th item examined is defective

$= 0$, otherwise.

Let $T = \sum_{i=1}^{n} X_i$. Consider the following statements.

I: $\frac{T}{n}$ is an unbiased estimator of p

II: $\frac{T(T-1)}{n(n-1)}$ is an unbiased estimator of p^2

Now state which of the following statement(s) is/are true?

A) I alone B) II alone C) Neither I nor II D) Both I and II

49. Suppose a random sample of size n is available from Cauchy distribution with p.d.f

$$f(x) = \frac{1}{\pi\left(1+(x-\mu)^2\right)}, x \in R, \mu \in R$$

Which of the following is a consistent estimator for μ ?

A) Sample median B) Sample mean

C) Sample range D) Sample mid-range

50. $X_1, X_2, ..., X_n$ is a random sample from a distribution with pdf $f(x) = \theta x^{\theta-1}; 0 < x < 1; \theta > 0$. Then

A) $\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ.

B) $\prod_{i=1}^{n} X_i$ is a sufficient statistic for θ.

C) $(1/n)\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ.

D) None of these

51. Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables with pdf

$$f(x) = \begin{cases} \frac{2\theta^2}{x^3}, & x > \theta; \theta > 0 \\ 0, & otherwise \end{cases}$$

then, the maximum likelihood estimator of θ is:

A) $[\prod_{i=1}^{n} X_i]^{3/2}$

B) $(1/n)\sum_{i=1}^{n} \ln X_i$

C) $\min(X_1, X_2, ..., X_n)$

D) $\max(X_1, X_2, ..., X_n)$

52. Let $X_1, X_2, ..., X_n$ be the random samples from a distribution with density:

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x > 0; \theta > 0 \\ 0, & otherwise \end{cases}$$

The moment estimator of θ is

A) $\sum_{i=1}^{n} \frac{X_i}{n}$

B) Sample median

C) $\frac{n}{\sum_{i=1}^{n} X_i}$

D) $(\sum_{i=1}^{n} X_i)^{1/n}$

53. Match list 1 with list 2 and select the correct answer using the code given below.

LIST 1	LIST 2
I. Barlett's Test	1. Probability of rejecting the null hypothesis when alternate hypothesis is true
II. ANOVA	2. Probability of rejecting the null hypothesis when it is true
III. Level of significance of a test	3. F- test
IV. Power of a test	4. t- test
	5. Chi square test

The correct match is:

	I	II	III	IV
A)	5	3	2	1
B)	5	4	2	1
C)	5	3	1	2
D)	4	5	1	2

54. Test statistic to test the significance of an observed multiple correlaion coefficient of a variable with k other variates in a random sample of size n follows:

A) An F distribution with (k, n-k) d.f

B) An F distribution with (k-1, n-k-1) d.f

C) An F distribution with (k, n-k-1) d.f

D) An F distribution with (k-1, n-1) d.f

55. Neyman Pearson lemma helps to obtain the most powerful test for testing:

A) A simple hypothesis against a composite alternative.

B) A composite hypothesis against a composite alternative.

C) A composite hypothesis against a simple alternative.

D) A simple hypothesis against a simple alternative.

56. If $X_1, X_2, ..., X_n$ are independent and identically distributed random variables following Normal distribution $N(\theta, \sigma^2)$; σ^2 is unknown. Then the likelihood ratio test for testing $H_0 : \theta = \theta_0 vs\ H_1 : \theta \neq \theta_0$ leads to:

A) One sided χ^2 test B) One sided students t test

C) Two sided students t test D) Two sided χ^2 test.

57. The degrees of freedom for the t-statistic in case of paired t-test based on N paires of observations is

A) N B) N-1 C) 2N-1 D) 2(N-1)

58. In a contingency table, the expected frequencies are computed under

A) Null hypothesis H_0 B) Alternative hypothesis H_1

C) Both H_0 and H_1 D) Neither H_0 nor H_1

59. A test that can be used in the non-parametric case instead of ANOVA is:

A) Sign test B) Wilcoxin

C) Kruskal-Wallis D) Mann-Whitney-U

60. In a Bayesian estimation problem of Poisson mean λ, a gamma prior is formulated. There is a sample of size n from a Poisson and the sample mean is $\bar{X}$, then:

A) The posterior distribution of λ is gamma

B) The posterior distribution of λ is Poisson

C) The posterior distribution of λ is Poisson-gamma

D) The prior is not a conjugate prior

61. Let X follows $b(n, p)$ and p follows $U(0,1)$ .Under squared error loss function, which of the following is not true?

A) Bayes estimator is $(x+1)/(n+2)$

B) Posterior distribution is beta

C) Posterior mean is $(x+1)/(n+2)$

D) Bayes estimator is posterior median

62. Let $X_1, X_2, ..., X_n$ be a random sample of size n from Normal population $N(\mu, 16)$. If a 95% confidence interval for μ is $(\bar{X} - 0.98, \bar{X} + 0.98)$, then the value of n is:

A) 4 B) 16 C) 32 D) 64

63. In SRSWR with usual notations, the standard error of the sample mean $\bar{y}$ is

A) $S\left(\frac{N-n}{Nn}\right)^{\frac{1}{2}}$ B) $S\left(\frac{N-1}{Nn}\right)^{\frac{1}{2}}$ C) $S\left(1-\frac{n}{N}\right)^{\frac{1}{2}}$ D) $\frac{S}{n}\left(1-\frac{1}{N}\right)^{\frac{1}{2}}$

64. With usual notation finite population correction is :

A) (N-1)/N B) (N-n)/N C) (N-n)/n D) 1-(1/n)

65. If the population is homogeneous, then which of the following is not preferred?

A) Simple Random Sampling B) Systematic Sampling
C) Stratified Sampling D) None of the above

66. The ratio estimator of population mean is unbiased if sampling is done according to

A) PPSWR B) PPSWOR C) SRSWR D) SRSWOR

67. The Midzuno-Sen scheme of sampling is a probability sampling scheme in which

A) Probability of selecting a unit is proportional to some measure of size of the unit.
B) Probability of selecting a unit is proportional to some measure of size of the units included in the sample.
C) Probability of selecting a sample is proportional to sum of measures of sizes of the units included in the sample.
D) All the possible samples are given an equal probability of selection.

68. Which type of experimental design doesn't use the principal of local control in measuring variation among the experimental units?

A) CRD B) RBD C) LSD D) BIBD

69. Consider a BIBD with parameters v, b, k, r, λ. A BIBD is said to be symmetric if

A) b=r, v=k B) b=v, r=k C) b=λ, v=b D) v=λ, b=k

70. In a RBD with r blocks, t treatments and one missing observation, the error degrees of freedom is:

A) $rt-r-t+1$ B) $rt-r$ C) $rt-r-1$ D) $rt-r-t$

71. Which of the following is/are linear contrasts?

i. $y_1-2y_2+2y_3$

ii. $3y_1+4y_2-7y_3$

A) i only B) ii only C) Both i and ii D) Neither i nor ii

72. Let $V=((v_{ij}))$ follows Wishart distribution $W_p(n, I_p)$, then v_{11} has

A) Chi-square distribution with n degrees of freedom
B) Chi-square distribution with (n-1) degrees of freedom
C) Non-central Chi-square distribution with n degrees of freedom
D) Non-central Chi-square distribution with (n-1) degrees of freedom

73. Let X follows multivariate normal $N_p(0,\Sigma)$ and A, C are matrices of order $p\times p$. Then $X'AX$ and $X'CX$ are independent if and only if

A) $AC=0$ B) $A\Sigma C=0$ C) $A\Sigma^{-1}C=0$ D) $A\Sigma^{-1}C=I$

74. If $X \sim N_p(\mu, \Sigma)$, then $E(X'X)$ is

A) $\Sigma^{-1}+\mu'\mu$ B) $\Sigma+\mu'\mu$
C) trace $(\Sigma^{-1}+\mu'\mu)$ D) trace $(\Sigma+\mu'\mu)$

75. Consider a Markov Chain with state space {1,2,3}and transition probability matrix $P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Then p_{11}^{99} is

A) 0 B) 1 C) $\frac{1}{2}$ D) None of these

76. Let $\{Z_r, r \geq 1\}$ be a sequence of independent and identically distributed random variables and let $X_n = Z_1 + Z_2 + \ldots + Z_n$, for $n \geq 1$. Then $\{X_n, n \geq 1\}$ is a Martingale if

A) $E(Z_1) < \infty$ B) $Var(Z_1) < \infty$

C) $E(Z_1) = 0$ D) $E(Z_1) = 1$

77. Let $\{X(t), t \geq 0\}$ be a stochastic process with stationary independent increments and let $X(0) = 0$ and $EX(1)\} = \mu$. Then $E\{X(t)\}$ is

A) 0 B) μ C) μt D) $\mu(t-s)$

78. If the number of closed sets in a Markov Chain is more than one, then the chain is

A) irreducible B) reducible C) ergodic D) non-ergodic

79. Match list 1 with list 2 and select the correct answer using the code given below.

LIST 1	LIST 2
I Walsh Price Index	1. The error caused by considering only the binary items.
II Formula Error	2. $\frac{P_{01} \times Q_{01}}{V_{01}} - 1$
III Homogeneity Error	3. The difference between Laspeyer's and Paasche's indices
IV Joint Error	4. Geometric cross formula
	5. Family budget method

The correct match is:

	I	II	III	IV
A)	2	3	5	1
B)	2	4	3	1
C)	5	3	1	2
D)	4	3	1	2

80. Consider the following statements.

I: Fisher's Ideal formula for index numbers does not satisfy circular test

II: Simple aggregate index does not satisfy unit test

State which of the following statement(s) is/are true?

A) I alone B) II alone

C) Neither I nor II D) Both I and II
